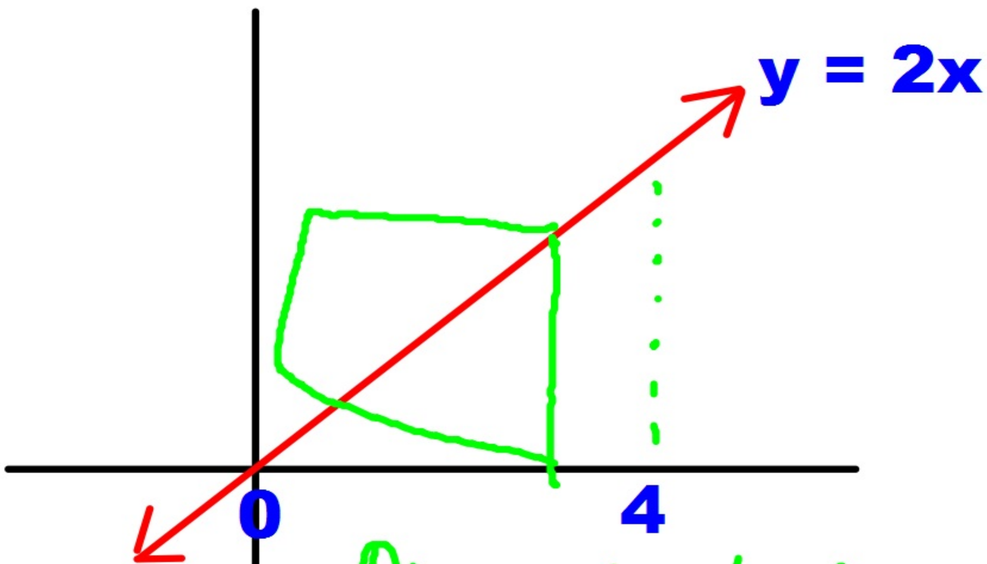
Section 7.3
Volumes

The volume of a solid of known integrable cross section area A(x) from x = a to x = b is the integral of A from a to b.

$$V = \int_a^b A(x)\, dx$$

Cross Sections

In the given situation, we can say that the cross sections are squares. Find the volume.



Dim = $2x$ by $2x$

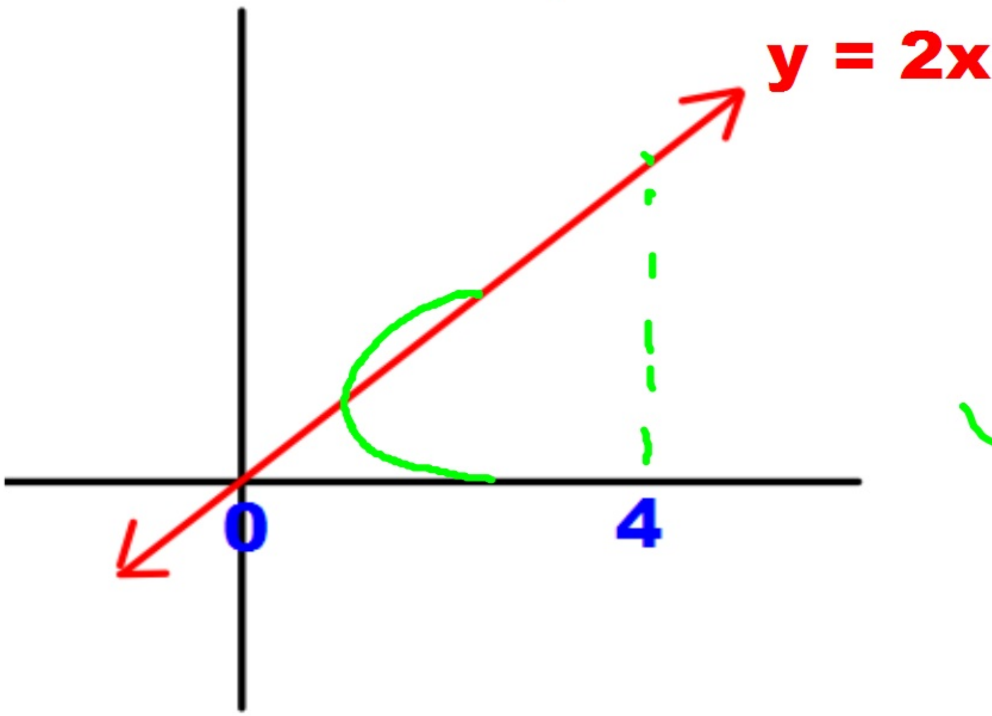
$A = 4x^2$

$$\int_0^4 4x^2\,dx =$$

In the given situation, we can say that the cross sections are semicircles. Find the volume.

$d = 2x$
$r = x$

$A = \frac{1}{2}\pi r^2$
$= \frac{1}{2}\pi (x)^2$

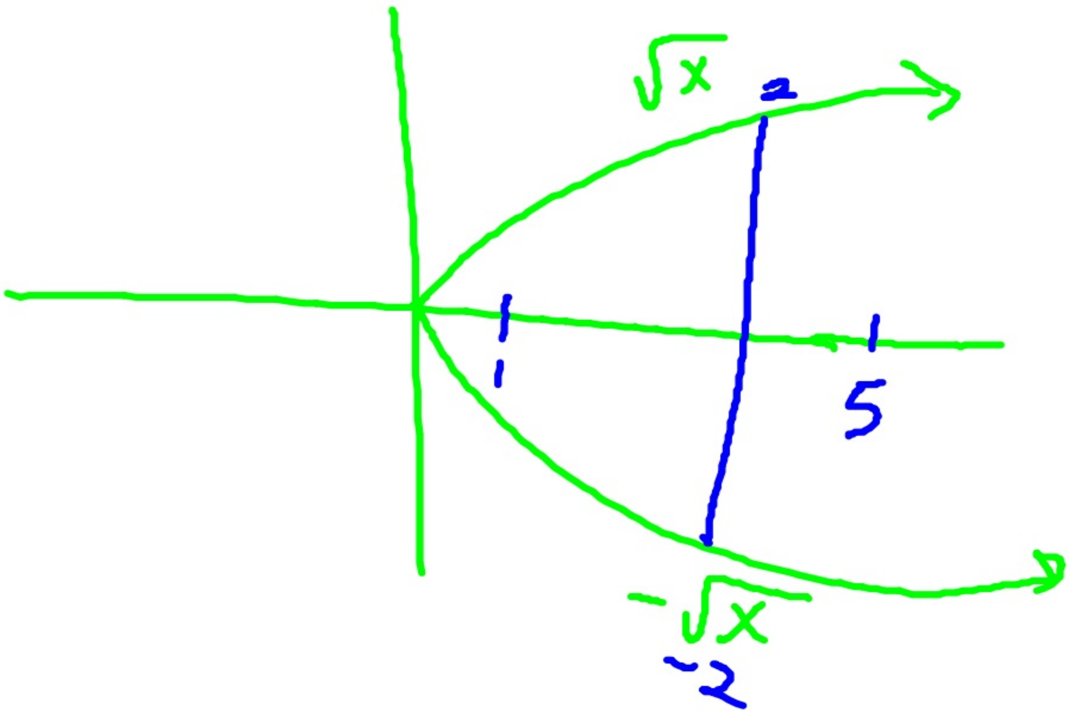


$$\int_0^4 \frac{1}{2}\pi x^2 \, dx = \frac{32\pi}{3}$$

Given the function, $x = y^2$, the cross sections are right triangles where the height is twice the base. Find the volume from 1 to 5.

$x = y^2$

$y = \pm\sqrt{x}$



$b = \sqrt{x} - (-\sqrt{x})$

$= 2\sqrt{x}$

$h = 4\sqrt{x}$

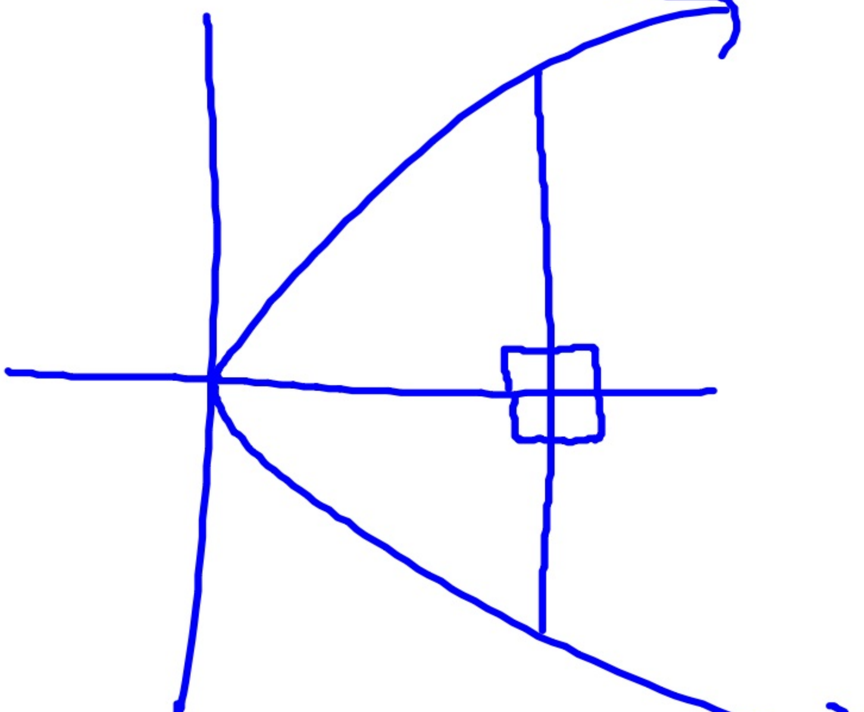
$A = \frac{1}{2}bh$

$= \frac{1}{2}(2\sqrt{x})(4\sqrt{x})$

$A = 4x$

$\int_1^5 4x\,dx =$

Given the function, $x = y^2$, the cross sections are right triangles where the height is 2/3 the base. Find the volume from 1 to 5.



$$b = 2\sqrt{x}$$

$$h = \frac{2}{3}\left(2\sqrt{x}\right)$$

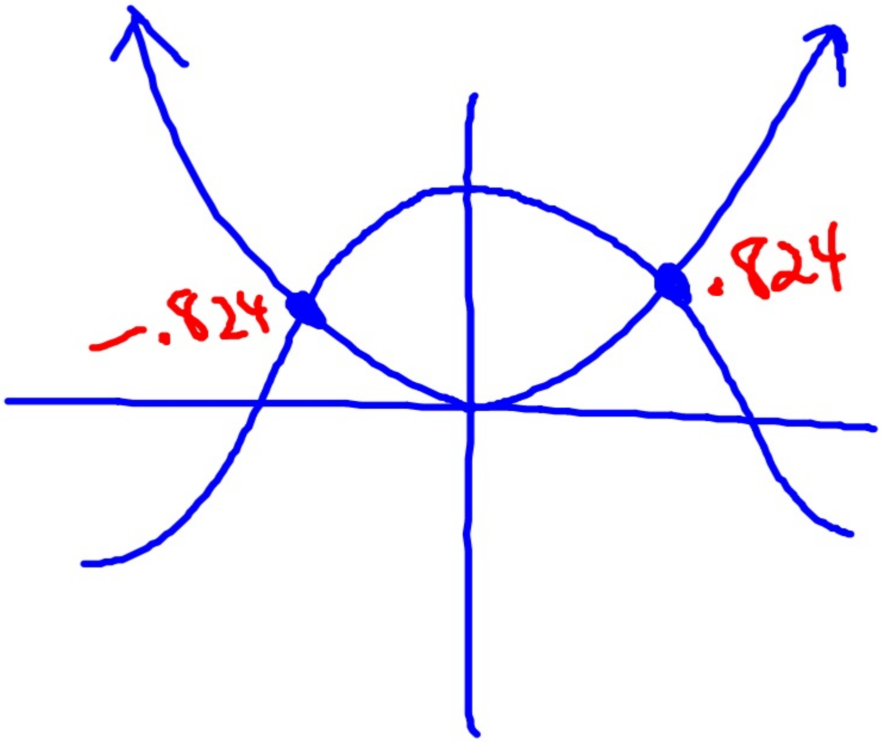
$$= \frac{4}{3}\sqrt{x}$$

$$A = \frac{1}{2}\left(2\sqrt{x}\right)\left(\frac{4}{3}\sqrt{x}\right)$$

$$\int^{5} \frac{4}{3} x \, dx$$

Given the two functions, $y = x^2$ and $y = \cos x$.

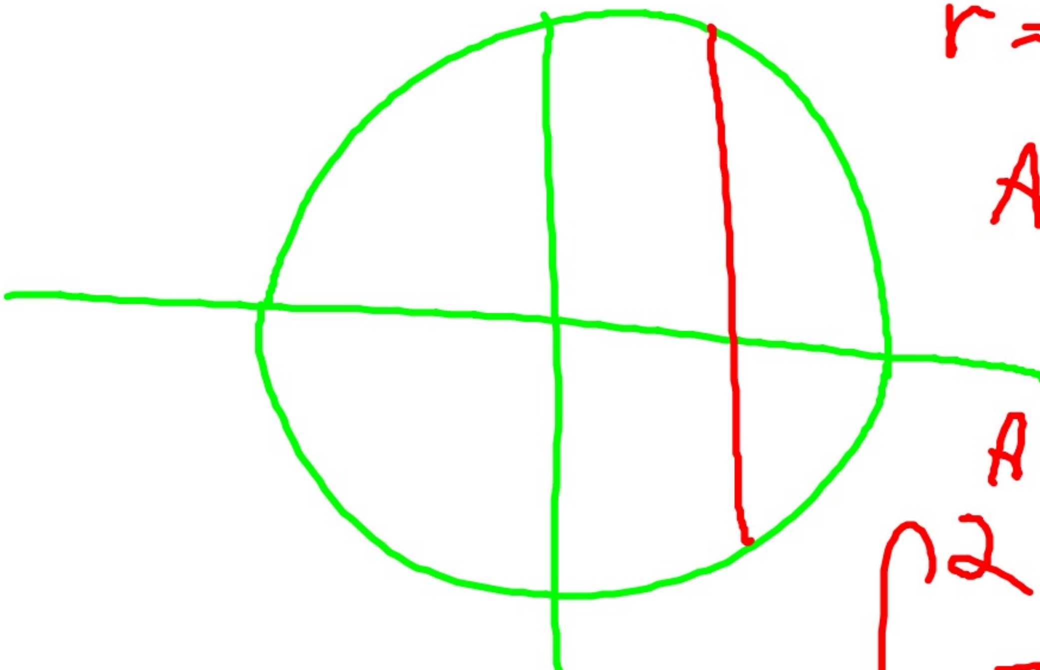
The cross sections are equilateral triangles.



Given $x^2 + y^2 = 4.$ Cross sections are circles.

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4 - x^2}$$



$$d = \sqrt{4 - x^2} - \left(-\sqrt{4 - x^2}\right)$$

$$= 2\sqrt{4 - x^2}$$

$$r = \sqrt{4 - x^2}$$

$$A = \pi r^2$$

$$= \pi\left(\sqrt{4 - x^2}\right)^2$$

$$A = \pi(4 - x^2)$$

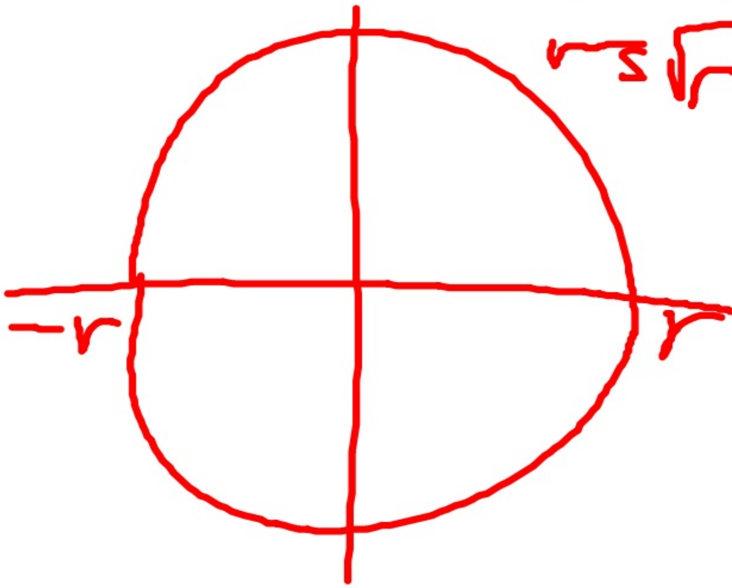
$$\int_{-2}^{2} \pi(4 - x^2)\,dx$$

Given $x^2 + y^2 = r^2$, Cross sections are circles.

$y = \pm\sqrt{r^2 - x^2}$

$d = 2\sqrt{r^2 - x^2}$

$r = \sqrt{r^2 - x^2}$



$A = \pi r^2$

$= \pi(r^2 - x^2)$

$$\int_{-r}^{r} \pi(r^2 - x^2)\,dx$$

$y = \sqrt{x}$ From 0 to 4.....Known as the Disk Method

Given $y = \sqrt{x}$ and $y = -2$

Washer Method

Page 406, numbers 2,4,39,43

A pyramid 3 m high has congruent triangular sides and a square base that his 3 m on each side. Each cross section of the pyramid parallel to the base is a square. Find the volume of the pyramid.

The region between the graph of $f(x) = 2 + x \cos x$ and the x-axis over the interval $[-2,2]$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis to form a solid. Find its volume.